Special termination, special finiteness reading note Spring 2024

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1 Overview

The aim of this note is try to prove [BCHM10, Lemma 4.4], which says global finiteness in dimension $(n-1)$ implies special finiteness in dimension n. And [BCHM10, Lemma 5.6] which shows special finiteness and existence of pl-flips in dimension n implies existence of minimal model in dimension n.

2 Global finiteness in dimension $n-1$ implies special finiteness in dimension n

In this section, we will prove the following result.

Theorem 1.

Let us first briefly sketch out the idea of the proof. We try to show that the MMP will terminate in some neighborhood of the divisor $S = \lfloor \Delta \rfloor$. The basic idea is that we try to use the adjunction technique, showing that the restriction terminates at S. An important step here is to try to show that the restriction of the weak log canonical model on S is still a log canonical model. Then we try to show that as long as the singularity is controlled, termination also happens in some neighborhood of S.

2.1 Restriction of log canononical model on boundary divisor

We try to prove the restriction of the weak log canonical model on S is still a weak log canonical model.

Lemma 2. Let $(X, \Delta = S + A + B)$ be a log smooth pair (over U), such that $S = \lfloor \Delta \rfloor$, $B \geq 0$ and A is general ample effective $\mathbb{Q}$-divisor. Assume that $(S, (\Delta - S)|_S)$ is terminal. Let $\phi : X \dashrightarrow Y$ be the weak log canonical model (X, Δ) over U, which does not contract S, with $T = \phi_* S$. The induce induced birational map $\tau : S \dashrightarrow T$ is weak log canonical model for (S, Ξ) for some divisor $A|_S \leq \Xi \leq \Theta = (\Delta - S)|_S$.

Moreover, if we write

$$(K_Y + \phi_* \Delta)|_T = K_T + \Psi,$$

then $\tau_* \Xi = \Psi$.

Let us briefly sketch out the idea of the proof.

Proof. □

2.2 Local uniqueness of canonical models near S when singularity on S being controlled

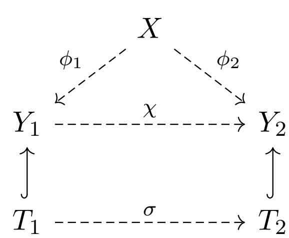
It's well known that log canonical model for a pair (X, Δ) is unique. If we varying the boundary Δ_i (with $S = \llcorner \Delta_i \lrcorner$), we still have local uniqueness of canonical model around S, when singularities are controlled.

Lemma 3. Let (X, Δ_i) (with $i = 1, 2$) be projective PLT pair over U, with $S = \llcorner \Delta_i \lrcorner$. Let

$$\phi_i : X \to Y_i,$$

be log canonical model of (X, Δ_i) over U which does not contract S. Let $\tau_i : S \dashrightarrow T_i$ be the restriction of ϕ_i on S and $(K_{Y_i} + T_i)|_{T_i} = K_{T_i} + \Phi_i$. If the following condition holds

- $\chi : Y_1 \dashrightarrow Y_2$ is small,
- The restriction $\sigma : T_1 \dashrightarrow T_2$ is isomorphism,
- The coefficients $\sigma^* \Psi_2 = \Psi_1$ and, for every component B of the support of $(\Delta_2 - S)$, we have $(\phi_{1*} B)|_{T_1} = \sigma^* \left((\phi_{2*} B)|_{T_2} \right)$,.



Then the induced birational map $Y_1 \dashrightarrow Y_2$ is isomorphism on some neighborhood of S (or say neighborhood of T_1 and T_2).

2.3 Proof of the theorem

References

[BCHM10] Caucher Birkar, Paolo Cascini, Christopher Hacon, and James Mckernan, *Existence of minimal models for varieties of log general type*, Journal of the American Mathematical Society **23** (2010), no. 2, 405–468.