A brief introduction of stacks Summer 2025

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The aim of this noe is to give a brief introduction to stacks. This note is based on the book [Alp25].

Contents

1 What is stack?

Grothendieck idea: find categorical object satisfies fpqc descend. In this note we will introduce the notion of stack by showing some examples: Moduli stack of curves, moduli stack of coherent sheaves and vector bundles, quotient stack and classifying stacks.

2 How to use 2-Yondeda Lemma in Moduli Theory?

Of course, here I'm working with the moduli stack rather than with the moduli space. For those of you who aren't familiar with stacks, don't worry: basically, all it means is that I'm allowed to pretend that the moduli space is smooth and that there's a universal family over it.-Joe Harris and Ian Morrison

3 Examples

3.1 Quotient stack and classifying stack

We then introduce the universal property of quotient stack.

3.2 Moduli stack of curves

3.3 Moduli stack of coherent sheaves and vector bundles

References

[Alp25] J. Alper, *tacks and moduli*, 2025, https://sites.math.washington.edu/~jarod/moduli.pdf.